$$c_1x + c_2x^{s_1(q-1)+1} + c_3x^{s_2(q-1)+1} \in \mathbb{F}_{q^2}[x] \quad 1 \le s_1, s_2 \le q,\ s_1 \ne s_2$$

Theorem 1. *[1] Let $a, b \in \mathbb{F}_{q^2}^*$. Then $ax + bx^q + x^{2q-1}$ is a permutation polynomial of $\mathbb{F}_{q^2}$ if and only if one of the following is satisfied*

1. $a = b^{1-q}$, $\mathrm{Tr}_{\mathbb{F}_q/\mathbb{F}_2}(b^{-1-q}) = 0$
2. $a \ne b^{1-q}$, $\frac{a}{b^2} \in \mathbb{F}_q$, $\mathrm{Tr}_{\mathbb{F}_q/\mathbb{F}_2}(a/b^2) = 0$, $b^2 + a^2b^{q-1} + a = 0$.

Theorem 2. *[2] Let $a, b \in \mathbb{F}_{q^2}^*$. Then $x + ax^{q(q-1)+1} + bx^{2(q-1)+1}$ is a permutation polynomial of $\mathbb{F}_{q^2}$ if and only if*

$$b(1 + a^{q+1} + b^{q+1}) + a^{2q} = 0$$

and

$$\begin{cases} \mathrm{Tr}_{\mathbb{F}_q/\mathbb{F}_2}\left(1 + \frac{1}{a^{q+1}}\right) = 0 & b^{q+1} = 1 \\ \mathrm{Tr}_{\mathbb{F}_q/\mathbb{F}_2}\left(\frac{b^{q+1}}{a^{q+1}}\right) = 0 & b^{q+1} \ne 1 \end{cases}$$

Theorem 3. *[3] Let $a, b \in \mathbb{F}_{q^2}^*$. Then $x + ax^{(1/4)q^2(q-1)} + bx^{(3/4)q^2(q-1)}$ is a permutation polynomial of $\mathbb{F}_{q^2}$ if and only if $a = b^{2-q}$ and $x^3 + x + a^{-1-q}$ has no root in $\mathbb{F}_q$.*

Theorem 4. *[4] Let $a, b \in \mathbb{F}_{q^2}^*$. $q \ge 2^3$. Then $x + ax^{(1/2)q(q-1)+1} + bx^{(1/2)q^2(q-1)+1}$ is a permutation polynomial of $\mathbb{F}_{q^2}$ if and only if $a = b^q$ and $\mathrm{Tr}_{\mathbb{F}_q/\mathbb{F}_2}(ab^q) = 0$.*

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